Bachelor in Physics
(Academic Year 2025-26)

Algebra			**Code**	800494	**Year**	1st	**Sem.**	2nd
Module	Basic Core	**Topic**	Mathematics		**Character**	Obligatory		

	Total	**Theory**	**Exercises**
ECTS Credits	7.5	4.5	3
Semester hours	69	39	30

Learning Objectives (according to the Degree's Verification Document)

To study and understand the following conceptual systems:

1. Linearity, linear independence and dimension
2. Linear applications: their matrix representation and the diagonalization problem.
3. The Geometry of spaces with scalar product. Symmetric and unitary operators.

Brief description of contents

Linear spaces and transformations. Euclidean spaces. Second degree curves.

Prerequisites

The Mathematics studied in High School.

Coordinator	Juan José Sanz Cillero		**Dept.**	FT
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Theory/Problems – Schedule and Teaching Staff

Group	Lecture Room	Day	Time	Professor	Period/ Dates	Hours	T/E	Dept.
B	7	Tu We Fr	9:00 – 10:30 9:30 – 11:00 11:00 – 13:00	Juan José Sanz Cillero	1st part	25	T/E	FT
				Rafael Hernández Redondo	2nd part	44		

T: Theory, E: Exercises

Office hours				
Group	**Professor**	**Schedule**	**E-mail**	**Location**
B	Juan José Sanz Cillero	M,J: 14:00-16:00 X: 11:00-13:00	jusanz02@ucm.es	02.327.0
	Rafael Hernández Redondo	M, X: 9:00-12:00	rafael.hernandez@fis.ucm.es	03.308.0

Syllabus

1.- PRELIMINARY:

1. Algebraic properties of real and complex numbers
2. Fundamental theorem of Algebra. Factorization of polynomials.
3. Systems of linear equations. Gauss elimination Method.
4. Matrices. Transposed matrix. Sum of matrices. Product of a scalar by a matrix.
5. Matrix product. Inverse matrix.

2.- VECTOR SPACES

1. Definition and examples of vector spaces. Linear combinations
2. Subspaces. Subspace generated by a set of vectors. Intersection and sum of subspaces.
3. Linear dependence and independence.
4. Bases. Dimension. Coordinates. Change of basis.
5. Direct sum of subspaces. Bases adapted to a direct sum.
6. Elementary operations in an ordered family of vectors.

3.- LINEAR MAPS, MATRICES AND DETERMINANTS

1. Definition and elementary properties of linear maps.
2. Nucleus and image of a linear map.
3. Injective, suprayective and bijective linear maps.
4. Matrix of a linear map. Change of basis.
5. The permutation group.
6. Determinants

4.- EIGENVALUES AND EIGENVECTORS

1. Eigenvalues and eigenvectors. Linear independence Theorem.
2. Characteristic polynomial.
3. Eigenspaces. Algebraic and geometric multiplicity. Diagonalization.
4. Invariant subspaces. Block diagonalization.

5.- SCALAR PRODUCT

1. Scalar product. Norm. Distance.
2. Parallelogram Identity. Polarization. Cauchy-Schwarz inequality. Triangular inequality.
3. Scalar product expression in a basis. Change of basis.
4. Orthogonality. Orthonormal bases. Gram-Schmidt method.
5. Orthogonal projection.

6.- LINEAR MAPS BETWEEN SPACES WITH SCALAR PRODUCT

1. Adjoint linear map. Elementary properties. Matrix representation.

2. Normal operators. Diagonalization of normal operators.
3. Self-adjoint and unitary operators in complex vector spaces.
4. Symmetric and orthogonal operators in real vector spaces. Rotations.

7.**- BILINEAL AND QUADRATIC FORMS**

1. Bilinear and quadratic forms in real spaces. Matrix representation. Change of basis.
2. Reduction of quadratic forms to sum of squares. Law of Inertia.
3. Factorizable real quadratic forms.
4. Positive definite quadratic forms. Sylvester's criterion.
5. Flat curves defined by second degree polynomials. Conics.

Bibliography

Basic:

- R. Larson, B. H. Edwards, D. C. Falvo, *Elementary Linear Algebra*, Houghton Mifflin Harcourt Publishing Company, 2009.
- D. C. Lay, *Linear Algebra and Its Applications (5th Edition)* Pearson Education Limited 2016.
- G. Strang, *Linear Algebra and its Applications*, Brooks Cole, International Edition, 2004.
- S. Lipschutz, Theory and Problems of Linear Algebra, Schaum's Outline Series. McGraw-Hill. 2004

Complementary:

- J. Arvesú, F. Marcellán, J. Sánchez, *Problemas Resueltos de Álgebra Lineal*. Thomson, 2005.
- M. Castellet, I. Llerena, C. Casacubierta, *Álgebra lineal y geometría.* Reverté, 2007.
- E. Hernández, *Álgebra y Geometría*, Addison Wesley/UAM, 1994.
- L. Merino, E. Santos, Álgebra Lineal, Editorial Paraninfo (2006).
- D. Poole, Álgebra Lineal: una introducción moderna, Thomson (2004).

ditionally, these open-access texts are recommended:

- Basic:
- https://www.cs.cornell.edu/courses/cs485/2006sp/LinAlg_Complete.pdf
- https://www.cliffsnotes.com/study-guides/algebra/linear-algebra
- https://www-labs.iro.umontreal.ca/~grabus/courses/ift6760_files/LANotes.lerner.pdf
- https://courses.physics.ucsd.edu/2009/Fall/physics130b/Essential_Linear_Algebra.pdf,
- https://cseweb.ucsd.edu/~gill/CILASite/
- Complementary: (In Spanish)
- http://jacobi.fis.ucm.es/marodriguez/notas_clase/algebra_AI_MAR.pdf

http://cms.dm.uba.ar/depto/public/Curso%20de%20grado/fascgrado2.pdf

Online Resources

Virtual Campus

Methodology
The following formative activities will be developed: - Theory lessons where the main concepts of the subject will be explained, including examples and applications (3 hours per week on average) - Practical classes of problems (2 hours per week on average) Students will be provided with a collection of problems prior to their resolution in the class. The teacher will receive in his office the students in the specified schedule of tutorials, in order to solve doubts, expand concepts, etc. It is highly recommended to attend these tutorials for a better use of the course. Students will be provided with exams of previous calls. It will be ensured that all the material of the subject is available to students through the Internet, in particular in the Virtual Campus.

Evaluation Criteria		
Exams	**Weight:**	75%
Midterm Exam: Yes	Eliminatory: No	Midterm exam weight: 40%
Midterm exam: - It will be about the content explained until that date and its structure will be similar to that of the final exam. $N_{Midterm}$ is the mark obtained on the midterm exam. - The contents evaluated in the partial exam may be subject to evaluation in the final exam. Final exam: - It will consist mainly of a series of problems on the contents explained during the whole course and of similar difficulty to those proposed in the collection of problems. Final mark for the Exams section, N_{Exam}: $N_{Exam} = \max \{ N_{Final} , 0.4\, N_{Midterm} + 0.6\, N_{Final} \}$ Minimum final exam mark to pass the course: $N_{Exam} \geq 4$ According to the Faculty Board agreement, at least 60% of the content of the first-year midterm and final exams must be shared by all groups.		
Other Activities	**Weight:**	25%
One or several of the following activities will be taken into account: - Problems and exercises delivered throughout the course individually or in groups during class time or outside it. - Participation in classes, seminars and tutorials. - Oral or written presentation of works. - Voluntary works.		

This grade will be retained for the extraordinary exam session. The final mark for this section will be $N_{OtherActiv}$ and will range from 0 to 10.
Final Mark
Final mark: $C_{Final} = \max\{ 0.75N_{Exam} + 0.25N_{OtherActiv} ,\ N_{Exam} \}$ Minimum final exam mark for weighting: $N_{Final} \geq 4$. The final mark criterion, as well as the mark corresponding to other activities, will be maintained in the exam of the extraordinary call.